The influence of heterogenous bargaining strengths on optimal wage negotiations

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Abstract

In this paper I show that - in contrast to the findings in the previous literature - substitutable workers can be better off negotiating in separate unions and complementary workers in one encompassing union. In addition, I find that results do not differ if two craft unions merge and negotiate as one bargaining agent or whether two separate craft unions negotiate with firms in one negotiation.

1 Introduction

Firms with heterogeneous workers are omnipresent. Mostly, firms cannot produce goods or services without heterogeneous and often complementary workers. Examples are manifold: manufacturing firms need at least blue and white collar workers, hospitals employ physicians, nurses, administration officers etc., airlines cannot conduct flights without pilots as well as flight attendants. Furthermore, each group of workers is not interchangeable. This complicates collective wage negotiations: how should workers organize themselves? Heterogeneous workers have to decide about two dimensions of organization. They must agree on the intra–union degree of heterogeneity and on the degree of centralization of the union. Concerning heterogeneity, workers have the choice of forming a union unifying workers along the lines of the particular profession (i.e., craft unions). Otherwise, they can establish comprehensive unions, where all workers in one firm or industry, independent of their profession, are organized within one encompassing union. With regard to centralization, workers have to determine if they want to form firm specific, industry wide, or even national unions.

In fact, most countries are characterized by a coexistence of different levels of heterogeneity as well as centralization. It is not obvious that an optimal level of the two dimensions exist. Moreover, the existing levels of centralization and heterogeneity of unions do not seem to be stable over time or countries and are influenced by several factors.

The literature on wage negotiations with heterogeneous workers (e.g., Horn and Wolinsky (1988) and Dowrick (1993)) is conclusive. They find that complementary workers should negoti-
ate separately. The reasons for this conclusion differ: Horn and Wolinsky take labor demand as given and show, that with separate negotiations workers do not take into account losses inflicted on other heterogeneous workers during strikes. Instead, Dowrick assumes an endogenous labor demand. If unions of complementary workers negotiate separately, they do not internalize the negative effect higher wages have on the employment of complementary workers. Thus, both papers conclude that individual wages are higher with separate negotiations. Substitute workers are better off within one union since firms cannot pit them against each other in that case.

These papers suggest that in equilibrium, substitutable workers are organized within one union, but no comprehensive unions exist. Maybe this is due to the fact that all models assume that bargaining strengths are equal for complementary groups of workers. The idea of my model is to analyze a situation where heterogeneous workers establish unions. Craft unions have heterogeneous and exogenous bargaining strengths. I draw comparisons between different levels of centralization and heterogeneity. Furthermore, I do not only compare wages but also union utility. I assume that union utility is increasing in wages and employment and thus, even if wages are lower, employment can be higher in equilibrium and this can be advantageous for unions. After all, I cannot unambiguously confirm previous results: First, in my model, it can be profitable for complementary workers to form an encompassing union. This is true for a strong craft union. When its bargaining strength is relatively strong in comparison to the other craft, it can be advantageous to form a coalition with a weak union even if these workers are complements. Wages as well as employment are higher with a coalition. It is also not always true that unions of substitutable workers should merge. Here, weak union benefits from a union merger. Again wages and employment are higher with a merger.

As a second result, I find that it does not matter whether two craft unions merge to one union and negotiate with firms or whether they both bargain in one negotiation round as antagonistic parties with firms. This is also in contrast to the claim that complementary unions should not merge. It simply does not matter if they merge or bargain separately in one negotiation round.

The theoretic literature on wage determination with unionized labor markets and heterogeneous (i.e., substitutable and complementary) workers is thin. The noteworthy papers are Horn and Wolinsky (1988), Dowrick (1993), and Gürtzgen (2003). Horn and Wolinsky assume a fixed rent which can be distributed between workers of different groups and the firm. They apply an extended version of the bargaining model of Rubinstein (1982) to obtain solutions. The process of wage determination is different to other papers: firms first choose employment, then workers decide on the patterns of unionization and in the last stage wage determination takes place. The firm chooses employment seeing through the next stages, knowing how the employment decision will affect the patterns of unionization and the wage negotiations. The authors choose this alternative setting compared to the wage–employment–negotiation literature to model a situation in which firms cannot change employment substantially without changing labor contracts with unions. Their main findings are straightforward: If the two types of workers are complements, they maximize their utility when they organize themselves in separate unions. If workers are substitutes in production they are better off if they form an encompassing union. This results from the fact that for substitutable workers one union is not better off withdrawing unilaterally and forming a separate union. The same is true for complementary workers: they are always better off in different unions since no distribution exist where not one groups would withdraw and form a separate union and obtain a higher payment.

Dowrick extends the Horn and Wolinsky paper in several ways. First, he takes product market competition into consideration. In his model, two firms compete in a market and higher wages in one firm influence wages in the other firm. Second, he sets up eight different cases

\footnote{In a non-oligopoly framework also Upmann (2008) finds under reasonable conditions that at least one group of workers has an incentive to negotiate separately. Firms always benefit from negotiations with a central union.}
which should capture different structural features of union and firm organization structure and bargaining locus. Third, strike payoffs which change wage negotiations are endogenous. Fourth, he assumes a symmetric Nash–Bargaining Solution to model wage negotiations. His main result is that a simple look at the negotiation level is not conclusive. Clear effects arise only when the level of organization—especially the one of the unions—is changed in the same way as the level of negotiation. Like Horn and Wolinsky, he finds that substitutable workers should form an encompassing union. However, Dowrick (1993) does not find that an encompassing employers’ association has systematic effects on wages. Even the bargaining locus has very ambiguous effects. But Dowrick (1993) shows that the claim to reduce the level of bargaining to firm negotiation to lower wages can be misleading: if unions and firms also decentralize their level of organization wages can rise. However, Dowrick only takes organization cases into consideration where unions are organized on craft level or merge to industry craft unions. He does not consider mergers of different craft unions to industry unions organizing different types of workers. This is the idea of Gürtzgen. She answers the question what happens if unions do not only merge “horizontally” like in Dowrick (1993) where cooperation takes places on the professional line. She also analyzes “vertical” mergers, where centralization across firm or industry lines occurs. Different from Dowrick, Gürtzgen assumes that unions can set the wages and firms employment. She refrains from modeling wage negotiations. Her main finding is that a ranking of wages according to the degree of centralization is not possible. A more decentralized bargaining does not necessarily lead to lower wages.

2 The Model

In my model unions and firms negotiate over wages under different wage negotiation regimes. Unions are upstream suppliers of workers in a labor market and firms compete in a product market. After firms and unions have agreed on wages, firms choose quantities and, therefore, employment in the product market. The game proceeds as follows:

1. Wage negotiations take place, and
2. firms set quantities in the product market.

In the downstream market two firms compete with profits of

$$\pi_i = (p - c_i)x_i, \quad (1)$$

where $i = 1, 2$ and $c_i$ are the marginal cost of production. To keep it simple, cost is solely labor cost. It is assumed that each firm needs two types of complementary workers (i.e., workers of different crafts) to produce the final output. For one unit of the end product, $N$ workers of type $n$ and $M$ workers of type $m$ are needed. Wages for these workers are $w_{in}$ and $w_{im}$, respectively. Thus, the cost for one unit of the end product is $c_i = Nw_{in} + Mw_{im}$ with $i = 1, 2$. The inverse demand function is

$$p = A - x_1 - x_2. \quad (2)$$

At most, four unions are active in the upstream labor market, $U_{1n}, U_{1m}, U_{2n}$, and $U_{2m}$. The number indicates the firm the union negotiates with, the latter the type of worker the union represents, for example $U_{1n}$ represents the union utility for all workers in firm 1 of craft $n$. Union utility increases with $w_{ij}$ and $x_i$, that is, more workers employed and higher wages increase union utility:

$$U_{im} = Mw_{im}x_i \quad (3)$$

$$U_{in} = Nw_{in}x_i \quad \text{with } i = 1, 2 \quad (4)$$

3
I analyze six union and firm organization structures (i.e., cases, henceforth). Each of these negotiations is solved applying an asymmetric Nash–Bargaining Solution (see Figure [1]).

I. Two negotiations in each firm, unions negotiate separately

In this decentralized setting, each firm negotiates with each craft union separately. Overall two negotiations per firm take place. Each craft union per firm tries to maximize its individual utility $U_{ij}$ and the firm maximizes $\pi_i$. During all negotiations, firms take into account that the agreement in one negotiation affects the outcome of the other.

II. One negotiation round for each firm, unions negotiate separately

In this case, each firm negotiates in one round with both craft unions active within the firm. The three parties, firm $i$, union $in$, and union $im$ sit together trying to maximize their individual utility ($\pi_i$, $U_{in}$ and $U_{im}$) choosing $w_{in}$ and $w_{im}$. Unions know that an increase in their wages decreases the number of workers in their firm, but also increases the number of workers in the other firm. This case is sometimes referred to as “single table bargaining” in the literature (See for example Dobson (1997)).

III. Two industry craft unions, each union bargains in one negotiation with both firms over industry-wide craft wages

Until now, each union has negotiated separately. Now I assume that workers of one craft working in different firms negotiate together. They try to maximize their joint utility by setting the industry craft wage $w_n$ or $w_m$. Two negotiations take place in the industry, where an industry craft union negotiates with both firms. During the negotiations each craft union internalizes how the wage affect wages of workers of the same craft in the other firm.

IV. Two industry craft unions, industry wide craft wages, one industry wide negotiation

In one centralized negotiation two industry wide craft unions negotiate with the two firms.

V. Two firm specific unions representing two different types of workers, one negotiation with each firm

In a first negotiation two distinct crafts in one firm bargain for the internal distribution of rents if they merge and form one firm specific union. Workers of different crafts in one union agree on a relative wage $\beta = w_{in}/w_{im}$. Afterwards the merged union bargains with its firm over absolute wages.

VI. One industry union negotiates with one employers’ association

This case is similar to V, but instead of two firm specific unions, one industry union representing both types of workers agree on a relative industry wage $\beta = w_n/w_m$. Afterwards this industry union bargains with one employers’ association.

The game is solved by backward induction: First the firms maximize their profits choosing $x_1$ or $x_2$

$$\max_{x_1} \pi_1 = (A - x_1 - x_2 - Nw_{1n} - Mw_{1m})x_1$$

$$\max_{x_2} \pi_2 = (A - x_1 - x_2 - Nw_{2n} - Mw_{2m})x_2,$$

resulting in

$$x_1 = \frac{A - 2(Mw_{1m} + Nw_{1n}) + Mw_{2m} + Nw_{2n}}{3}$$

$$x_2 = \frac{A - 2(Mw_{2m} + Nw_{2n}) + Mw_{1m} + Nw_{1n}}{3}.$$
Figure 1: The negotiation cases
Afterwards firms and unions negotiate about wages, taking the employment decision into account.

I. Two negotiations per firm, unions negotiate separately

The negotiations are modeled using a Nash–Bargaining Solution. Overall, four wage negotiations take place between unions and firms,

\[ \begin{align*}
N_{in} &= U_{in}^a \pi_i^c \\
N_{im} &= U_{im}^b \pi_i^c
\end{align*} \]

for \( i = 1, 2 \),

where \( a, b, \) and \( c \) are the bargaining strengths of unions and of the firm respectively. Maximizing the Nash–Bargaining Solutions with respect to \( w_{in} \) and \( w_{im} \) yields the following results:

\[ \begin{align*}
w_{in} &= \frac{a A(b + 2c)}{2N(2ab + 3bc + 3ac + 4c^2)} \\
w_{im} &= \frac{b A(b + 2c)}{2M(2ab + 3bc + 3ac + 4c^2)}
\end{align*} \]

Plugging this back into (1), (2), (3), (5), and (6), yields the equilibrium results for quantities, price, profits, union utility, and welfare which can be found in the Appendix.

II. One negotiation per firm, unions negotiate separately

Now, only two wage negotiations take place

\[ N_i = U_{in}^a U_{im}^b \pi_i^c \] for \( i = 1, 2 \).

This Nash–Bargaining Solution is maximized choosing \( w_{in} \) and \( w_{im} \) and the four wages are

\[ \begin{align*}
w_{in} &= \frac{a A}{(3a + 3b + 4c)N}, \text{ and} \\
w_{im} &= \frac{b A}{(3a + 3b + 4c)M}
\end{align*} \]

Again, the results for quantities, price, profits and union utility can be found in the Appendix.

III. Two industry craft unions, each union bargains in one negotiation with both firms over industry wide craft wages

Here, negotiations take place on the industry level. Workers of the same craft in different firms form an encompassing union. Craft union utility is

\[ \begin{align*}
U_n &= U_{1n} + U_{2n} = Nw_n (x_1 + x_2) \\
U_m &= U_{1m} + U_{2m} = Mw_m (x_1 + x_2).
\end{align*} \]

Both industry wide craft unions negotiate with both firms separately over industry wide craft wages \( w_n \) and \( w_m \).

\[ \begin{align*}
N_n &= U_{in}^a \pi_1^c \pi_2^c \\
N_m &= U_{im}^b \pi_1^c \pi_2^c
\end{align*} \]

Maximizing the Nash–Bargaining Solution with respect to \( w_n \) and \( w_m \) yields,

\[ \begin{align*}
w_n &= \frac{a A(b + 4c)}{N(3ab + 8bc + 8ac + 16c^2)} \\
w_m &= \frac{b A(a + 4c)}{M(3ab + 8bc + 8ac + 16c^2)}
\end{align*} \]
IV. Two industry craft unions, one industry wide negotiation, industry wide craft wages

Both firms and both industry craft unions negotiate over wages $w_n$ and $w_m$ in one industry wide negotiation,

$$N = U_n^a U_m^b \pi_1 \pi_2.$$

These industry negotiations result in industry wide wages of $w_n$ and $w_m$. The final wages they agree on are

$$w_n = \frac{aA}{2N(a + b + 2c)}$$

$$w_m = \frac{bA}{2M(a + b + 2c)}.$$

V. Two firm specific unions representing two different types of workers, one negotiation with each firm

Different than before, negotiations do not take place simultaneously. Since workers of different crafts form an encompassing union, they agree on the distribution of rents first. Afterwards, each encompassing unions negotiates with its firm. Solving the game backward, I first maximize the Nash–Bargaining product of the union–firm negotiation

$$N_{1i} = U_i^{(a+b)} \pi_i^c$$

for $i = 1, 2$ and afterwards the intra–union negotiation,

$$N_{2i} = U_{in}^a U_{im}^b$$

for $i = 1, 2$.

For simplicity, I assume that the union bargaining strength when the two crafts negotiate together equals the sum of their individual bargaining strength alone. Solving this model backwards, during the union–firm negotiations, both parties take relative wages as given. In the intra union negotiation, crafts negotiate over relative wages. The equilibrium results are equal to the ones found in case II.

VI. One industry union negotiates with one employers’ association

Again, wage negotiations are not simultaneous. Solving backwards, I first maximize the industry wide Nash–Bargaining product with one union and one employers’ association, taken relative wages $\beta$ as given,

$$N_1 = (U_{n1} + U_{m1} + U_{n2} + U_{m2})^{2(a+b)} (\pi_1 + \pi_2)^{2c}.$$

Afterwards, in a second round, the two industry craft unions distribute their rents

$$N_2 = U_n^a U_m^b.$$

Maximizing this expression with respect to relative wages results in the equilibrium wages. Here results are equal to case IV.

3 Negotiation Results: Why Do Different Negotiation Frameworks Result in Equal Wages?

Proposition 1

1. Case II and V are equal for a bargaining strength of the encompassing union of $a + b$. 


2. Cases IV and VI are equal for a bargaining strength of the encompassing union of \(2(a+b)\).

At first glance, it is surprising that wages are equal in cases II and V. In case II, the two unions and the firm negotiate together in one bargaining round as three conflicting parties. However, this results in the same wages as in case V. Here, craft unions form an encompassing union and agree on allocation quotas for their joint rent in a first negotiation. Afterwards, they bargain as one “strong” union representing different crafts within the firm. It does not matter whether the unions merge and bargain as one party (one union framework, henceforth), or negotiate separately but in one negotiation round with the firm (i.e., one negotiation framework).

It is quite amazing, in fact, that an individual utility maximization in case II yields the same results as a joint utility maximization in case V. In the second case the different craft unions do not internalize the negative external effect their wage has on workers of the other craft in the same firm. This effect is internalized in case V, the share of the burden of the external effect each craft has to bear is determined through negotiations. However, results are the same. To show why one union and one negotiation framework result in equal wages, I use a simpler model:

Assume three parties negotiate for a cake \(A\). The size of the cake is fixed and does not change due to the negotiation. This is different to the original setting—unions and firms can influence the size of the rent through the wage—but this is simpler and does not influence the results. The parties negotiation strengths are \(a\), \(b\), and \(c\), respectively. They all try to maximize utility and utility is defined as the part of the cake they receive,

\[
V_1 = \alpha A, \quad V_2 = \beta A, \quad V_3 = (1 - \alpha - \beta) A,
\]

where \(\alpha\), \(\beta\), and \((1 - \alpha - \beta)\) are the shares of the cake the parties receive. In the one negotiation framework the asymmetric Nash–Bargaining Solution for the three parties is

\[
N = V_1^a V_2^b V_3^c.
\]

Maximizing the Nash–Bargaining Solution with respect to the shares \(\alpha\) and \(\beta\) yields:

\[
\frac{dN}{d\alpha} = \frac{a}{V_1} \frac{dV_1}{d\alpha} + \frac{b}{V_1} \frac{dV_1}{d\alpha} + \frac{c}{V_1} \frac{dV_3}{d\alpha} = 0
\]

\[
\frac{\alpha}{a} = 1 - \alpha - \beta
\]

\[
\frac{dN}{d\beta} = \frac{a}{V_1} \frac{dV_1}{d\beta} + \frac{b}{V_1} \frac{dV_1}{d\beta} + \frac{c}{V_1} \frac{dV_3}{d\beta} = 0
\]

\[
\frac{\beta}{b} = 1 - \alpha - \beta
\]

As can be seen nicely here, shares are distributed relative to the bargaining strengths. Solving for \(\alpha\) and \(\beta\) results in equilibrium shares of

\[
\alpha = \frac{a}{a+b+c} \quad \text{and} \quad \beta = \frac{b}{a+b+c}.
\]

Hence, equilibrium utilities for the three parties are:

\[
V_1 = \frac{a}{a+b+c} A, \quad V_2 = \frac{b}{a+b+c} A, \quad \text{and} \quad V_3 = \frac{c}{a+b+c} A.
\]

\[\footnote{In my model, the quantity reaction of the firms in respond to a wage altering influence the cake size but this effect is equal in both frameworks and can therefore be neglected.}\]
What is different in a one union framework? Here, two parties form a coalition and divide the rent they receive from a negotiation with the third party in a separate negotiation. Let us assume that the first and the second party form a coalition and their joint utility is $V_{1+2} = (\alpha + \beta)A$. When this coalition negotiates with the third party, assume that their bargaining strength in this negotiation round is simply the sum of their individual strengths. Then the asymmetric Nash–Bargaining Solution is,

$$ N = V_{1+2}^{(\alpha+\beta)}V_3^{\gamma}. $$

The derivative with respect to $(\alpha + \beta)$ is

$$ \frac{dN}{d(\alpha + \beta)} = \frac{a + b}{V_{1+2}} \frac{dV_{1+2}}{d(\alpha + \beta)} + \frac{c}{V_3} \frac{dV_3}{d(\alpha + \beta)} = 0 $$

Again, the shares are relatively distributed to their negotiations strengths. The share of the cake the coalition gets is

$$ \alpha + \beta = \frac{a + b}{a + b + c}. $$

Stated differently, the first and second party receives a slice with a size of $B := (a+b)/(a+b+c)A$. In another negotiation round, they have to distribute this slice $B$. The Nash–Bargaining Solution for this negotiation is

$$ N = V_1^a V_2^b. $$

For simplicity, rewrite the utility functions as $V_1 = \gamma B$, $V_2 = (1-\gamma)B$. The size of the slice is not influenced through the negotiations, it is fixed at this stage. Maximizing the Nash–Bargaining Solution yields the well known result:

$$ \frac{dN}{d\gamma} = \frac{a}{V_1} \frac{dV_1}{d\gamma} + \frac{b}{V_1} \frac{dV_2}{d\gamma} = 0 $$

$$ \frac{a}{\gamma} = \frac{b}{1-\gamma} $$

The share of the slice $B$ the first party receives is $\gamma = a/(a+b)$ which yields an overall utility for the first party of

$$ V_1 = \frac{a}{a+b} B = \frac{a}{a+b} \frac{a+b}{a+b+c} A = \frac{a}{a+b+c} A $$

As can be seen in this expression, party 1 receives a share of $a/(a+b)$ of the slice $B$ which is equal to

$$ \frac{a}{a+b+c} = \frac{a}{a+b+c} $$

of cake $A$.

This result is driven by the fact that I assume that the bargaining strengths after forming a coalition are the sum of the individual bargaining strengths. Otherwise the cumulative bargaining strength $a+b$ would not be simply cancel out in Eq. 7. When I loosen that assumption and assume that the cumulative strength is higher than the individual, not surprisingly, the coalition would receive a higher share than negotiating separately and the third party would suffer.

To sum up, it does not matter whether union negotiate as one party in a negotiation (i.e., one union framework) or as two conflicting parties in one negotiation (i.e., one negotiation
framework) as long as the bargaining strength of the coalition is the sum of the individuals. The negative external effect of higher wages on the different crafts is a mirror–inverted effect. Higher wages of \( m \) type workers reduces employment of \( n \) type workers and vice versa. Due to a Nash–Bargaining Solution which maximizes the whole cake \( A \), depending on both union utility and firm profits, this effect is internalized as it reduces cake size and does not lead to higher or lower wages than in case \( V \) where this effect is directly internalized through the unions’ utility function.

Exactly the same argumentation is true for wages in case \( IV \) and \( VI \). Here as well, the formation of a coalition—also on firm side—makes no difference to a single negotiation. The negotiations yields the same results as long as the coalition bargaining strength is the sum of the individual ones.

This result is empirically supported by Machin, Stewart, and van Reenen (1993) and Metcalf (1993). They show for British workers that wages do not differ between the one union framework and one negotiation framework. However, a theoretical explanation for this fact was missing in their papers.

4 Comparing the Negotiation Cases

The aim of my paper is to find out which of the negotiation cases firms and unions prefer and which is welfare maximizing. Therefore, with heterogeneous bargaining strengths, I compare quantities, wages, profits, union utility, and welfare for different cases. As established above, I can reduce six regimes to four, as it turned out that two of them lead to the same results. First, I will present the results for quantities and profits in the different cases, and afterwards, wages, union utility, and welfare comparisons are shown.

To understand the changes due to different negotiation regimes it is helpful to calculate derivatives to better understand the effects. First, a change in wages changes the quantities produced,

\[
\frac{dx_i}{dw_{in}} < 0, \quad \frac{dx_i}{dw_{im}} < 0 \quad (8)
\]

\[
\frac{dx_i}{dw_{jn}} > 0, \quad \frac{dx_i}{dw_{jm}} > 0. \quad (9)
\]

Derivatives with respect to wages show that quantities decrease if the own wages increase, and quantities increase if wages of the competitor increase. This should not be surprising as long as wages are a cost for firms and an increase in own cost reduces quantities produced, and higher cost of competitors increase own production. However, since the model is a linear Cournot model, profits are squared equilibrium quantities, and the derivative signs of quantities correspond with the derivative signs of profits. Second, a change of wages has an indirect effect on wages of the other workers through employment,

\[
\frac{dw_{in}}{dx_i} \frac{dx_i}{dw_{im}} < 0, \quad \frac{dw_{in}}{dx_i} \frac{dx_i}{dw_{jn}} > 0, \quad \frac{dw_{in}}{dx_i} \frac{dx_i}{dw_{jm}} > 0.
\]

If wages of complementary workers in the same firm \( w_{im} \) increase, this is a negative externality for wages \( w_{in} \) because quantity \( x_i \) is reduced and ceteris paribus less workers of type \( n \) are needed. Additionally, through the product market effects, higher wages of workers in the other firm, \( w_{jn} \) and \( w_{jm} \) increase the cost for firm \( j \), therefore production of firm \( i \) increases and due to that wages \( w_{in} \).
Comparing the different negotiation cases, most results are driven by the fact that workers organize themselves in such a way that they internalize (or not) the external effects that their wages have on other groups. Stated differently, when complementary workers of one firm form an encompassing union they do internalize the effect of higher wages on employment of complementary workers within the same firm. This lowers, ceteris paribus, wages within the firm. In contrast, when workers of different firms form one union, they internalize the positive effect higher wages have on employment of workers in the other firm and this ceteris paribus increases wages. Analogously, firms can internalize the positive effect of higher wages (and therefore lower quantities) on each other. Thus, an employers’ association lowers wages.

4.1 Quantities and Firm Profits

With these general effects established above in mind, the order of the quantities of different regimes is plausible:

**Proposition 2** \( x_{i}^{II} > x_{i}^{I} \geq x_{i}^{IV} > x_{i}^{III} \).

The highest production occurs when each firm negotiates with its two craft unions in one negotiation. In this case \( II \), the product of the union utilities and firm profits is maximized in the negotiation. Therefore, the negative external effect that higher wages of \( w_{in} \) have on \( w_{im} \) (and vice versa) through lower employment are internalized.\(^3\) This internalization of the negative external effect leads ceteris paribus to lower wages and higher quantities. Additionally, firms do not negotiate in one round and for that reason they do not internalize the positive external effect a lower quantity would have on the competitor. Hence, production is highest in case \( II \).

The lower quantity in case \( I \) is obvious. When the two crafts negotiate separately, they do not keep in mind the negative effects of higher wages on the other type of workers in the same firm. They demand higher wages, employment in the firm is reduced and this results compared to \( II \) in lower quantities.

In case \( IV \) one industry wide negotiation takes place. All four unions take the negative and positive external effects into account. The effect of this on quantities is ambiguous: higher wages have negative external effects on quantities produced in that firm, but positive effects on quantities of the other firm. However, firms do internalize the positive effect of lower quantities on their competitor and thus reduce production. Overall quantities are lower in case \( IV \) than in case \( I \), only for unions with the same bargaining strength (i.e., \( a = b \)), one can show that quantities are the same for \( I \) and \( IV \).

In case \( III \), the lowest quantities are produced. Here, industry craft unions negotiate with firms separately. Unions do not take the negative effect of higher wages on workers of the same firm but different crafts into consideration and employment and quantities are reduced compared to \( IV \).

Unsurprisingly, the order of prices is opposite to that of quantities. Since in this Cournot model profits are equal to the squared equilibrium quantities, we also know:

**Proposition 3** Firms prefer \( \pi_{i}^{II} > \pi_{i}^{I} \geq \pi_{i}^{IV} > \pi_{i}^{III} \).

This is true independent of the values of the bargaining strengths of firms and unions and the reasons are similar to the explanation for quantities: profits are just squared equilibrium quantities. Profits are highest for firms if they negotiate only with firm specific unions and especially when the different crafts take the negative external effects of higher wages into account. Even when the unions do not consider the negative effects but again the firms negotiate separately

\(^3\) For a discussion why this effect is internalized, see section [ ]
on firm–level with their unions this leads to high profits. Unambiguously, profits for the firms are lower if unions of different crafts merge.

4.2 Wages

The ranking of wages in the different cases is much more puzzling than the ranking of quantities and profits.

Proposition 4

1. Wages are ordered \( w_{j}^{III} > w_{j}^{II} \) and \( w_{j}^{IV} > w_{j}^{II} \) for \( j = n, m \).

2. The comparison between \( w_{j}^{I} \) and \( w_{j}^{II} \), \( w_{j}^{I} \) and \( w_{j}^{III} \), \( w_{j}^{III} \) and \( w_{j}^{IV} \) is ambiguous and depends on \( a, b, \) and \( c \).

3. For \( a > b, w_{j}^{IV} > w_{j}^{I} \) and \( w_{m}^{IV} > w_{m}^{I} \), analogously for \( b > a, w_{n}^{IV} < w_{n}^{I} \) and \( w_{m}^{IV} < w_{m}^{I} \).

\( \text{For } a = b, w_{n}^{IV} = w_{n}^{I} \) and \( w_{m}^{IV} = w_{m}^{I} \).

Independent of the bargaining strength, wages in case II are always lower than in III and IV. This is not surprising, due to the fact that in case II the workers of one craft in one firm take the negative effects of higher wages on the workers of the other craft into account. This lowers wages. Instead, in III and IV, industry wide craft unions were formed, taking the positive externality into account which leads to higher wages, but not to the negative externality on complementary workers.

The relationships between \( w_{j}^{I} \) and \( w_{j}^{II} \), \( w_{j}^{I} \) and \( w_{j}^{III} \), \( w_{j}^{III} \) and \( w_{j}^{IV} \) is even more complicated. They strongly depend on the bargaining strengths of the various parties. Thus, it is helpful to model the effects influencing wages for \( w_{1n} \) and \( w_{1m} \) in a more formal way4. Let us assume union utility is defined as \( U_{1n} \) and \( U_{1m} \), and profits are \( \pi_{i} \) and \( \pi_{j} \).

4. First, I compare cases I and II. I argue above that in case I, where the two craft unions negotiate separately, they do not internalize the negative effects of higher wages on complementary workers in the same firm and demand for higher wages. Hence, wages \( w_{1n}^{II} \) should be higher than \( w_{1n}^{I} \). However, this is not true in general. To show why, I compute the implicit function to obtain \( w_{1n}^{I} \) for \( n \) in I by logarithmically deriving the corresponding Nash–Bargaining Solution \( N = U_{1n}^{I} \pi_{1}^{c} \).

\[
\frac{dN}{dw_{1n}} = \frac{a}{U_{1n}^{I} dw_{1n}} + \frac{c}{\pi_{1}^{c} dw_{1n}} \leq 0
\]

(10)

This defines the wage function \( w_{1n}^{I}(w_{1m}, w_{2n}, w_{2m}) \). Here, two effects occur. A variation of wage \( w_{1n} \) has a direct effect on \( U_{1n} \) and it also influences the firm. For further analysis, it is helpful to remind the behavior of the wage function. As can be shown easily,

\[
\frac{dw_{1n}^{I}}{dw_{1m}} < 0, \frac{dw_{1n}^{I}}{dw_{2n}} > 0, \frac{dw_{1n}^{I}}{dw_{2m}} > 0.
\]

Put another way, the wage function \( w_{1n}^{I} \) is downward sloping in higher wages of complementary workers in firm 1 and upwards sloping in wages of workers in firm 2. Plotting the wage function \( w_{1n}^{I}(w_{1m}) \) keeping \( w_{2n}, w_{2m} \) constant is a decreasing function (see Figure 2), \( w_{1n}^{I}(w_{2n}) \) and

4The effects for the other wages can be established analogously.

5See Davidson (1988) for details with this approach, as well as Dowrick (1993) and Gürtzgen (2003).

6For a discussion whether this is a best response function see fn.10 in Davidson (1988). Dowrick (1993) calls the same functions “reaction function”, however, to avoid misconceptions I called these functions “wage functions” henceforth.
\( w_1^{I*}(w_2) \) are increasing. The intersection point of all four wage functions yields equilibrium wages.

In case II, \( w_1 \) is negotiated applying \( U_{1n}^a \) and this results in
\[
\frac{dN}{dw_{1n}} = \frac{a}{U_{1n}} \frac{dU_{1n}}{dw_{1n}} + \frac{b}{U_{1m}} \frac{dU_{1m}}{dw_{1m}} + \frac{c}{\pi_1} \frac{d\pi_1}{dw_{1n}} = 0
\]

with \( w_1^{II*}(w_1, w_2, w_2) \) as the wage function. Again, the slopes are as expected: \( dw_1^{II*}/dw_{1n} < 0 \), \( dw_1^{II*}/dw_{2m} > 0 \). Wages \( w_1^{II*} \) increase with increasing wages in firm 2 and decrease with higher wages of the complementary workers \( w_1 \). Here, three effects affect the wage function: One is the direct effect on union utility of the workers under consideration. The next is an indirect effect on the union utility of complementary workers \( U_{1m} \) through quantities and, finally, the influence on firm profits. This is strictly negative as long as higher wages reduce firm profits.

To compare \( w_1^{I*} \) and \( w_1^{II*} \), I evaluate (10) at \( w_1^{II*} \) and obtain,
\[
S_1 := - \frac{b}{U_{1m}} \frac{dU_{1m}}{dw_{1m}} > 0
\]

This implies that the wage function \( w_1^{I*} \) lies on the right hand side of \( w_1^{II*} \). With upward sloping wage functions this would always result in higher equilibrium wages. Equivalently, one can compare wages of workers of type \( m \), \( w_1^{I*} \), which solves
\[
\frac{dN}{dw_{1m}} = \frac{b}{U_{1m}} \frac{dU_{1m}}{dw_{1m}} + \frac{c}{\pi_1} \frac{d\pi_1}{dw_{1m}} = 0
\]

and \( w_1^{II*} \) solving
\[
\frac{dN}{dw_{1m}} = \frac{a}{U_{1m}} \frac{dU_{1m}}{dw_{1m}} + \frac{b}{U_{1m}} \frac{dU_{1m}}{dw_{1m}} + \frac{c}{\pi_1} \frac{d\pi_1}{dw_{1m}} = 0
\]
The outward shift is here

$$S_2 := - \frac{a}{U_{1n}} \frac{dU_{1n}}{dw_{1m}} > 0.$$  \hspace{1cm} (13)

Whether the wages are higher or lower with downward sloping wage functions depends on the shifts $S_1$ and $S_2$ and thus $a$ and $b$. As you can see in Figure 2, the outward shift with downward sloping demand functions can result in lower wages. Wages $w_{1m}$ are lower than $w_{2m}$.

What does it mean intuitively? This effect may appear awkward at first glance. The reason why one intuitively assumes that $w_{1n}$ is higher than $w_{2n}$ is simply because $w_{2n}$ is found taking the negative indirect effect on complementary workers into account which reduces $w_{1n}$. Let’s call this effect wage effect. Obviously it is negative. How strong this negative effect is, depends on the value of $b$ (see Eq. 12) that is the bargaining strength of the complementary workers. In addition, there is an employment effect. During wage negotiations of complementary workers of type $m$, they also take into account the negative effect they have on worker type $n$. This effect becomes large with a high bargaining strength $a$ (see Eq. 13). For a high $a$, $w_{1m}$ becomes low compared to $w_{2m}$, this leads to a higher employment and increases the employment of workers of both types. This also yields higher wages of type $n$ workers. To sum it up, the negative wage effect can be outweighed by a positive employment effect when the bargaining strengths are sufficiently different. This can be seen for workers of type $n$ in Figure 2 contrary to that, the employment effect does not outweigh the wage effect for workers of type $m$.

Theoretically, the crucial point is that when one assumes symmetric union utility functions, one way to establish this result are varying bargaining strengths. In the literature this result is new. Davidson compares substitutable workers with upward sloping best response functions where no positive employment effect occurs. Dowrick assumes complementary workers and wage negotiations. However, he assumes symmetric bargaining strengths which shifts the wage functions outwards by the same value and the employment effect never outweigh the wage effect.
Finally, Gürtzgen also assumes complementary workers but abstracts from wage negotiations and, therefore, this effect does not occur.

One could also think about symmetric bargaining strengths but asymmetric union utility functions or union sizes to establish similar results. The value of the outward shift depends not only on union strength but also on union utility and the cross derivative (see Eqs. 12 and 13). If these values are very distinct, similar effects with lower wages in case I can occur. Examples one can think of are unions of very different sizes or wage oriented vs. employment oriented unions.

The relation between \( w_{III} \) and \( w_{IV} \) is comparable to \( w^I \) vs. \( w^{II} \). To find \( w_{III}^{IV*}(w_{1n}, w_{2n}, w_{2m}) \) one has to solve

\[
\frac{dN_n}{dw_{1n}} = \frac{a}{U_n} \frac{dU_n}{dw_{1n}} + \frac{c}{\pi_1} \frac{d\pi_1}{dw_{1n}} + \frac{c}{\pi_2} \frac{d\pi_2}{dw_{1n}} \equiv 0, \tag{14}
\]

and for \( w_{IV}^{IV*}(w_{1m}, w_{2n}, w_{2m}) \)

\[
\frac{dN_n}{dw_{1n}} = \frac{a}{U_n} \frac{dU_n}{dw_{1n}} + \frac{b}{U_m} \frac{dU_m}{dw_{1n}} + \frac{c}{\pi_1} \frac{d\pi_1}{dw_{1n}} + \frac{c}{\pi_2} \frac{d\pi_2}{dw_{1n}} \equiv 0. \tag{15}
\]

Again, one finds a direct effect of a wage increase for \( w_{III}^{IV} \) and \( w_{IV}^{IV} \), but an indirect effect only for \( w_{III}^{IV} \) and the slopes of wage functions are as expected: in both cases wages increase in \( w_{2n} \) and \( w_{2m} \). They always decrease with \( w_{1m} \). To find the direction of the shift, I calculate \( \frac{dU_m}{dw_{1n}} \) and get

\[ S_3 := \frac{b}{U_m} \frac{dU_m}{dw_{1n}} < 0. \]

Thus, \( w_{III}^{IV*}(w_{1m}, w_{2n}, w_{2m}) \) shifts to the right of \( w_{IV}^{IV*}(w_{1m}, w_{2n}, w_{2m}) \)

The intuitive explanation is analogous to the comparison between \( I \) and \( II \). At first sight it seems obvious that wages in case \( III \) are higher compared to \( IV \) as long as the negative external effect on complementary workers is internalized under \( IV \). Put differently, the negative wage effect only occurs in case \( IV \) and one expects lower wages there. However, when the two union strengths are sufficiently different, I find again a positive employment effect. For workers of type \( n \) this means that when their bargaining strength \( a \) is high and bargaining strength \( b \) of type \( m \) is low, the wage effect is modest. In addition, workers of type \( m \) are confronted with a larger negative wage effect, their wages are low and this has a positive external employment effect on workers of type \( n \).

Different explanations are necessary for the comparison of wages in cases \( I \) and \( III \). I compute \( w_{IV}^I(w_{1m}, w_{2n}, w_{2m}) \) and \( w_{III}^{IV*}(w_{1m}, w_{2n}, w_{2m}) \) again (see \( 11 \) and \( 14 \)). To find the direction of the shift, I evaluate \( \frac{dU_n}{dw_{1n}} \) at \( w_{IV}^I \) using \( 11 \). Taking advantage of the symmetry of the firms in the model, the firm effects are equal and the shift is therefore,

\[
a \left( \frac{1}{U_n} \frac{dU_n}{dw_{1n}} - \frac{1}{U_m} \frac{dU_m}{dw_{1n}} \right) \gtrless 0.
\]

\( ^7 \)It can also be established that for a comparison between \( w_{III}^{IV}(w_{1n}, w_{2n}, w_{2m}) \) and \( w_{IV}^{IV*}(w_{1n}, w_{2n}, w_{2m}) \), the shift is \( S_4 := a/U_n * dU_n/dw_{1m} < 0 \). Again, \( w_{IV}^{IV*}(w_{1m}, w_{2n}, w_{2m}) \) lies at the right side of \( w_{III}^{IV}(w_{1n}, w_{2n}, w_{2m}) \).
Whether this term is positive or negative is not obvious. Thus I do not know the direction of the shift. But even with a non-ambiguous sign, due to the complementary workers and the dependence of the shifts on $a$, $b$ and $c$ I have to calculate the equilibrium wages to see which effect dominates. For a wide parameter space, wages are higher in case III but with a low $a$ and high $b$ and a low $c$, $w^I_{1n}$ can be higher than $w^I_{1n}$. This means, that when the workers of type $n$ are weak and the complementary workers are strong, it can lead to higher wages if they negotiate separately with their firm and do not form an industry wide union and negotiate with both firms together.

Finally, the comparison of wages between case I and IV is puzzling. For the same bargaining strength union wages under I and IV are equal. Wages do not differ whether each firm specific craft union negotiates separately with its firm, or whether two industry specific craft unions are formed and negotiate in one round with both firms an industry wide wage. In case I unions do not take into account any positive or negative external effects. In case IV unions take all positive and the negative effects under consideration. Additionally, in case I only one firm takes part in each negotiation, in case IV negative and positive external effects between firms are internalized.

To compare the wage functions, I calculate \ref{eq:15} at $w^I_{1n}$ ($w_{1n}, w_{2n}, w_{2m}$) using \ref{eq:11}. Benefiting from the symmetry of the firms in the model, firm effects are equal and the shift equals

$$\frac{a}{U_n} \frac{dU_n}{dw_{1n}} + \frac{b}{U_m} \frac{dU_m}{dw_{1n}} - \frac{a}{U_{1n}} \frac{dU_{1n}}{dw_{1n}} \leq 0$$

or

$$a \left( \frac{1}{U_n} \frac{dU_n}{dw_{1n}} - \frac{1}{U_{1n}} \frac{dU_{1n}}{dw_{1n}} \right) + b \left( \frac{1}{U_m} \frac{dU_m}{dw_{1n}} \right) \leq 0$$

First of all, the indirect effect on the wages of the complementary workers does only appear under IV and not under I and this negative effect reduces ceteris paribus $w^IV$ but not $w^I$. The
size of this effect depends on $b$.

Second the substitutable worker effect is always positive. This is due to the well known fact, that $\frac{1}{U_n} \cdot \frac{dU_n}{dw_n}$ internalizes the positive effect higher wages have on substitutable workers. This leads ceteris paribus to a $w^{IV}$ that should be higher than $w^I$. Which effect, the complementary worker or the substitutable worker effect predominates, depends on $a$ and $b$. For $a = b$ these two effects outweigh each other and wages are equal under $I$ and $IV$. However, for $a > b$ the substitutable worker effect dominates and $w^I$ is lower than $w^{IV}$, if $b > a$ the complementary worker effect dominates and $w^{IV} < w^I$.

Here the employment effect due to asymmetric shifts of wage functions of workers of complementary crafts does not matter: To see this, I calculate the shift for $w_{im}$:

$$\frac{a}{U_n} \cdot \frac{dU_n}{dw_{im}} + b \left( \frac{1}{U_m} \cdot \frac{dU_m}{dw_{1m}} - \frac{1}{U_{1m}} \cdot \frac{dU_{1m}}{dw_{1m}} \right) \leq 0,$$

and both shifts are plotted in Figure 3. They simply reinforce each other. A right shift for workers of type $n$ results in a left shift for workers of type $m$ and vice versa.

### 4.3 Unions

Union utility increases in wages and employment. Thus, the ranking of union utility in different cases is a combined effect of quantities and wages. As shown above, results for quantities have a clear sorting whereas the ranking of wages is ambiguous. Due to that, the ranking for union utility is also ambiguous.

**Proposition 5**

1. $U_j^{IV} > U_j^{II}$ for all $j = n, m$.

2. Comparing $U_j^I$ and $U_j^{III}$, $U_j^I$ and $U_j^{III}$, $U_j^{III}$ and $U_j^{IV}$, and $U_j^{II}$ and $U_j^{III}$ with $j = n, m$ depends on bargaining strengths $a$, $b$ and $c$.

3. For $a > b$ $U_j^{IV} > U_j^I$ and $U_m > U_m^I$, analog for $b > a$ $U_j^{IV} < U_j^I$ and $U_m < U_m^I$, if $a = b$ $U_j^{IV} = U_j^I$.

Obviously, union utility is higher in case $IV$ than in case $II$. This due to the fact that wages are higher in case $IV$. However, employment is highest in case $II$, but this cannot outweigh lower wages. Therefore, unions prefer one industry wide negotiation about industry wages with industry craft unions to one negotiation per firm with firm specific craft unions.

A little bit astonishing, a comparison between case $II$ and $III$ does not always lead to a higher union utility in case $III$ even if wages are always higher in that case. The higher employment in case $II$ outweighs higher wages if union strength is high and complementary union and firm strength low. Here, unions favor lower wages and higher employment to higher wages and lower employment. Stated in economic terms, if the union is strong, it is advantageous to stay as a firm specific craft union and negotiate on firm level than to merge with substitutable workers in the other firm and negotiate over industry wages. This is a result different to the ones published before which always suggest that substitutable workers should form one encompassing union. This leads to higher wages, indeed, but can be disadvantageous due to lower employment.

The comparison of wages between $I$ and $II$, $I$ and $III$, and $III$ and $IV$ is ambiguous. I found an employment effect which leads to higher wages of complementary workers which was

---

*Plugging in the terms yields $an/(A - mw_n + mw_m)$ which is always positive.*
not obvious at first sight. However, this employment effect on wages is intensified: not only wages are higher, but also overall employment. Employment was always higher in case II than case I and sometimes also wages. Therefore, the parameter space where unions prefer II to I is larger than the parameter space where wages are higher under II than I. The same is true for I and III and II and IV.

What does that mean economically? When a union has a high bargaining strength the union wants to negotiate in one negotiation with complementary workers and the firm instead of two separate negotiations where both unions negotiate separately. This is different from the well known fact that complementary workers should negotiate separately. Case II is equivalent to case V where unions of complementary workers merge. These joint negotiations always lead to higher employment and sometimes also to higher wages. However, only the strong unions prefer the single table negotiations, the weak one always prefers the separate. For the firms the situation is easier. They always prefer joint negotiations yielding a larger quantity.

The comparison between III and IV is similar. In case III the two industry unions negotiate separately with the two firms. In case IV on industry wide negotiation between the two firms and the two unions take place. As long as quantities are always higher with one industry wide negotiation, firms prefer that scenario. However, also strong unions prefer this setting. Their wages are higher with a joint negotiation and also employment is higher. Only the weak union always prefers the separate negotiations. Again, it is not true to state that it is always in the interest of workers not to negotiate with complementary workers. With heterogeneous bargaining strength, this is only true for workers with a lower bargaining strength.

Comparing cases I and III shows that firms always favor higher production in case I. That was not surprising since in case I each firm negotiates alone with each of its crafts and in case III each firm has to agree in one negotiation round with its competitor and one industry craft union. This was the worst situation for firms where substitutable worker form a coalition but not the firms. However, sometimes also weak unions can prefer I over III. Here the result is again that it is not always in the interest of a union to negotiate together with substitutable workers.

The last comparison is between I and IV. Here, higher employment in case I does not matter. The union always prefers the higher wages; the higher employment cannot outweigh lower wages. So, the ranking of union utility is equal to the ordering of wages.

4.4 Welfare

Welfare is defined here as the sum of consumer surplus, profits, and union utility.

\[ W = \frac{1}{2} (A - p)(x_1 + x_2) + \pi_1 + \pi_2 + U_{1n} + U_{2n} + U_{1m} + U_{2m} \]

Proposition 6 \[ W^{II} > W^{I} \geq W^{IV} > W^{III} \].

Welfare is in its ordering exactly equal to the ordering of quantities and profits. This may be surprising at first glance. However, the driving force is that consumer surplus and firm profits are highly correlated. Firm profits are highest with the highest employment and quantity produced. This is equivalent to low prices and a high consumer surplus. This correlation between firm profits and consumer surplus is higher than the lower union utility in some cases. This result simplifies economic advices. As long as consumer surplus and producer surplus is highly correlated, the enforcement of the negotiation regime firms prefer also enlarges consumer surplus and overall welfare.
5 Conclusion

In my model, I draw comparisons with horizontal as well as vertical union mergers and allow for different bargaining strengths. I do not only sort the bargaining cases by wage levels, but by union utility levels. For this to be feasible I have to presuppose a specific union utility function.

My results are twofold: First, I can show that results do not differ whether unions (or firms) form an encompassing union (or employers’ association) and negotiate internally over the distribution of rents or whether they negotiate as conflicting parties in a Nash–Bargaining Solution. This is only true if the bargaining strength of the encompassing union (or employers’ association) equals the sum of the individual bargaining strengths. For wage bargains this implies that no difference occur between a one union or a one negotiation framework.

Second, in contrast to the literature I cannot verify that it is always in the interest of complementary workers to be organized in different unions and to form encompassing unions for substitutable workers. Once one assumes different union strengths wages can be higher when substitutable workers negotiate alone or when complementary workers form an encompassing union. However, this higher wage effect is reinforced due to higher employment when substitutable workers negotiate separately or complementary workers together. For the simple fact that union utility increases in wages and employment it is sometimes utility enhancing for complementary unions to negotiate together and for substitutable unions to negotiate alone.

Thus, different bargaining strengths complicate the comparison. Unions have different incentives to form encompassing union or to negotiate together and this strongly depends on their relative bargaining strength.

The model differs from the previous literature in multiple ways: The main idea is that bargaining agents do not necessarily have the same bargaining strength during negotiations. With regard to wage negotiations, incentives for firms to form an employers’ association or for unions to form encompassing unions differ due to heterogeneous bargaining strengths. Dowrick only deals with horizontal union mergers and in his model unions and firms always have a symmetric bargaining strength. However, Gürtzgen extends Dowrick’s model to vertical union mergers but she refrains from wage negotiations and thus heterogeneous bargaining strength.

References


Appendix

A Different Negotiation Cases

A.1 Two Negotiations per Firm, Unions Negotiate Separated (I)

\[
\begin{align*}
    w_{in}^{I} &= \frac{aA(b + 2c)}{2(N(2ab + 3bc + 3ac + 4c^2))}; \\
    w_{im}^{I} &= \frac{bA(a + 2c)}{2(M(2ab + 3bc + 3ac + 4c^2))}; \\
    x_{i}^{I} &= \frac{A(b + 2c)(a + 2c)}{3(2ab + 3bc + 3ac + 4c^2)}; \\
    p^{I} &= \frac{A(4ab + 5ac + 5bc + 4c^2)}{3(2ab + 3ac + 3bc + 4c^2)}; \\
    U_{in}^{I} &= \frac{aA^2(b + 2c)^2(a + 2c)}{6(2ab + 3bc + 3ac + 4c^2)^2}; \\
    U_{im}^{I} &= \frac{bA^2(a + 2c)^2(b + 2c)}{6(2ab + 3bc + 3ac + 4c^2)^2}; \\
    \pi_{i}^{I} &= \left(x_{i}^{I}\right)^2 = \left(\frac{A(b + 2c)(a + 2c)}{3(2ab + 3bc + 3ac + 4c^2)}\right)^2; \\
    W^{I} &= \frac{2A^2(b + 2c)(a + 2c)(5ab + 7bc + 7ac + 8c^2)}{9(2ab + 3bc + 3ac + 4c^2)^2};
\end{align*}
\]

A.2 One Negotiation Per Firm, Unions Negotiate Separated (II)

\[
\begin{align*}
    w_{in}^{II} &= \frac{aA}{(3b + 4c + 3a)N}; \\
    w_{im}^{II} &= \frac{bA}{(3b + 4c + 3a)M}; \\
    x_{i}^{II} &= \frac{2A(a + b + 2c)}{3(3b + 4c + 3a)}; \\
    p^{II} &= \frac{A(5b + 4c + 5a)}{3(3b + 4c + 3a)}; \\
    U_{in}^{II} &= \frac{2aA^2(a + b + 2c)}{3(3b + 4c + 3a)^2}; \\
    U_{im}^{II} &= \frac{2bA^2(a + b + 2c)}{3(3b + 4c + 3a)^2}; \\
    \pi_{i}^{II} &= \left(x_{i}^{II}\right)^2 = \left(\frac{2A(a + b + 2c)}{3(3b + 4c + 3a)}\right)^2; \\
    W^{II} &= \frac{4A^2(a + b + 2c)(7b + 8c + 7a)}{9(3b + 4c + 3a)^2};
\end{align*}
\]
A.3 Two Industry Craft Unions, Each Union Negotiates in One Negotiation with Both Firms (III)

\[
\begin{align*}
    w_n^{III} &= \frac{aA(b + 4c)}{N(3ab + 8bc + 8ac + 16c^2)}; \\
    x_i^{III} &= \frac{A(b + 4c)(a + 4c)}{3(3ab + 8bc + 8ac + 16c^2)}; \\
    U_n^{III} &= \frac{2aA^2(b + 4c)^2(a + 4c)}{3(3ab + 8bc + 8ac + 16c^2)^2}; \\
    \pi_i^{III} &= (x_i^{III})^2 = \left(\frac{A(b + 4c)(a + 4c)}{3(3ab + 8bc + 8ac + 16c^2)}\right)^2; \\
    W^{III} &= \frac{8A^2(b + 4c)(a + 4c)(2ab + 5bc + 5ac + 8c^2)}{9(3ab + 8bc + 8ac + 16c^2)^2}.
\end{align*}
\]

A.4 Two industry craft unions, one industry wide negotiation (IV)

\[
\begin{align*}
    w_n^{IV} &= \frac{aA}{2N(a + b + 2c)}; \\
    x_i^{IV} &= \frac{A(a + b + 4c)}{6(a + b + 2c)}; \\
    U_n^{IV} &= \frac{aA^2(a + b + 4c)}{6(a + b + 2c)^2}; \\
    \pi_i^{IV} &= (x_i^{IV})^2 = \left(\frac{A(a + b + 4c)}{6(a + b + 2c)}\right)^2; \\
    W^{IV} &= \frac{A^2(a + b + 4c)(5a + 5b + 8c)}{18(a + b + 2c)^2}.
\end{align*}
\]

B Proofs

B.1 Proposition 2

To see that \(x_1^{II} > x_1^{IV} > x_1^{III}\), a comparison of the quantities is necessary, and yields

\[
\begin{align*}
    x_1^{II} - x_1^{IV} &= \frac{1}{3} \frac{Aab (a + b + 4c)}{(2ab + 3ac + 3bc + 4c^2)(3a + 3b + 4c)} > 0; \\
    x_1^{I} - x_1^{IV} &= \frac{1}{6} \frac{Ac(a - b)^2}{(a + b + 2c)(2ab + 3ac + 3bc + 4c^2)} > 0 \text{ if } a \neq b; \\
    x_1^{IV} - x_1^{III} &= \frac{1}{6} \frac{Aab (a + b + 8c)}{(a + b + 2c)(3ab + 8ac + 8bc + 16c^2)} > 0.
\end{align*}
\]

B.2 Proposition 3

As long as profits in all regimes are squared quantities, the results for wages apply to profits.
**B.3 Proposition 4**

1. To see that \( w_{j}^{III} > w_{j}^{II} \) and \( w_{j}^{IV} > w_{j}^{II} \) for \( j = n, m \) it is sufficient to show that \( w_{j}^{III} - w_{j}^{II} > 0 \) and \( w_{j}^{IV} - w_{j}^{II} > 0 \).

\[
\begin{align*}
    w_{n}^{III} - w_{n}^{II} &= \frac{Aa (4ac + 8bc + 3b^2)}{N (3ab + 8ac + 8bc + 16c^2) (3a + 3b + 4c)} > 0 \\
    w_{m}^{III} - w_{m}^{II} &= \frac{Ab (8ac + 4bc + 3a^2)}{M (3ab + 8ac + 8bc + 16c^2) (3a + 3b + 4c)} > 0 \\
    w_{n}^{IV} - w_{n}^{II} &= \frac{Aa (a + b)}{2N (3a + 3b + 4c) (a + b + 2c)} > 0 \\
    w_{m}^{IV} - w_{m}^{II} &= \frac{Ab (a + b)}{2M (3a + 3b + 4c) (a + b + 2c)} > 0
\end{align*}
\]

2. The relation between \( w_{j}^{I} \) and \( w_{j}^{II} \), \( w_{j}^{III} \) and \( w_{j}^{IV} \) is ambiguous.

\[
\begin{align*}
    w_{n}^{II} - w_{n}^{I} &= \frac{Aab (a - 3b - 4c)}{2N (3a + 3b + 4c) (2ab + 3ac + 3bc + 4c^2)} \leq 0 \\
    w_{m}^{II} - w_{m}^{I} &= \frac{Aab (b - 3a - 4c)}{2M (2ab + 3ac + 3bc + 4c^2) (3a + 3b + 4c)} \leq 0 \\
    w_{n}^{III} - w_{n}^{II} &= \frac{Aa (2b^2c - ab^2 - 8abc - 8ac^2)}{2N (3ab + 8ac + 8bc + 16c^2) (2ab + 3ac + 3bc + 4c^2)} \leq 0 \\
    w_{m}^{III} - w_{m}^{II} &= \frac{Ab (2a^2c - ba^2 - 8abc - 8ac^2)}{2M (3ab + 8ac + 8bc + 16c^2) (2ab + 3ac + 3bc + 4c^2)} \leq 0 \\
    w_{n}^{IV} - w_{n}^{III} &= \frac{Aab (2b - a + 4c)}{2N (a + b + 2c) (3ab + 8ac + 8bc + 16c^2)} \leq 0 \\
    w_{m}^{IV} - w_{m}^{III} &= \frac{Ab (2b - a + 4c)}{2M (a + b + 2c) (3ab + 8ac + 8bc + 16c^2)} \leq 0
\end{align*}
\]

3. For \( a > b, \) \( w_{j}^{IV} > w_{j}^{I} \) and \( w_{j}^{III} > w_{j}^{IV} \), analogously for \( b > a, \) \( w_{j}^{IV} < w_{j}^{I} \) and \( w_{j}^{III} < w_{j}^{IV} \). For \( a = b, \) \( w_{j}^{IV} = w_{j}^{I} \) and \( w_{j}^{III} = w_{j}^{IV} \).

\[
\begin{align*}
    w_{n}^{IV} - w_{n}^{I} &= \frac{Aa (b + c) (a - b)}{2N (2ab + 3ac + 3bc + 4c^2) (a + b + 2c)} \leq 0 \\
    w_{n}^{I} - w_{n}^{IV} &= \frac{Ab (a + c) (a - b)}{2M (2ab + 3ac + 3bc + 4c^2) (a + b + 2c)} \leq 0
\end{align*}
\]

**B.4 Proposition 5**

1. A subscription shows that \( U_{j}^{IV} > U_{j}^{II} \) for all \( j = n, m \).

\[
\begin{align*}
    U_{n}^{IV} - 2U_{n}^{II} &= \frac{A^2a (a + b) (2ab + 12ac + 12bc + a^2 + b^2 + 16c^2)}{6 (3a + 3b + 4c)^2 (a + b + 2c)^2} > 0 \\
    U_{m}^{IV} - 2U_{m}^{II} &= \frac{A^2b (a + b) (2ab + 12ac + 12bc + a^2 + b^2 + 16c^2)}{6 (3a + 3b + 4c)^2 (a + b + 2c)^2} > 0
\end{align*}
\]
2. Comparing \( U^I_j \) and \( U^{III}_j \), \( U^I_n \) and \( U^{III}_n \) and \( U^I_m \) and \( U^{III}_m \) with \( j = n, m \) depends on bargaining strengths \( a, b \) and \( c \) is ambiguous.

\[
U^{III}_n - U^I_n = \frac{A^2 ab \Delta_1}{6 (2ab + 3ac + 3bc + 4c^2)^2 (3a + 3b + 4c)^2} \leq 0
\]

\[
\Delta_1 = -9ab^3 + 7a^3b - 32ac^3 + 12a^3c - 128bc^3 - 18b^3c - 2a^2b^2 + 28a^2c^2 - 84b^2c^2 - 64c^4 - 72abc^2 - 48ab^2c + 14a^2bc
\]

\[
U^{III}_m - U^I_m = \frac{A^2 ab \Delta_2}{6 (2ab + 3ac + 3bc + 4c^2)^2 (3a + 3b + 4c)^2} \leq 0
\]

\[
\Delta_2 = -9a^3b - 18a^3c - 2a^3b^2 - 48a^2bc - 84a^2c^2 + 7ab^3 + 14ab^2c - 72abc^2 - 128ac^3 + 12b^3c + 28b^2c^2 - 32bc^3 - 64c^4
\]

\[
2U^I_{in} - U^{III}_{in} = \frac{A^2 a \Delta_3}{3 (2ab + 3ac + 3bc + 4c^2)^2 (3a + 8ac + 8bc + 16c^2)^2} \leq 0
\]

\[
\Delta_3 = -512a^6 + a^3b + 9a^3b^4 - 384a^2b^5 - 32a^3c^4 + 256b^6 + 56b^4c^3 - 400a^2b^2c^3 - 28a^2b^2c^2 - 65ac^6 + 384ab^2c^2 + 112ab^2c^3 + 464b^3c^2 - 768a^2b^4c + 10a^2b^4c - 80a^3b^3c - 4a^3b^3c
\]

\[
2U^I_{im} - U^{III}_{im} = \frac{A^2 b \Delta_4}{3 (3ab + 8ac + 8bc + 16c^2)^2 (2ab + 3ac + 3bc + 4c^2)^2} \leq 0
\]

\[
\Delta_4 = -512b^6 + a^3b + 9a^3b^4 - 384a^2b^5 - 32a^3c^4 + 256b^6 + 56b^4c^3 - 400a^2b^2c^3 - 28a^2b^2c^2 - 65ac^6 + 384ab^2c^2 + 112ab^2c^3 + 464b^3c^2 - 768a^2b^4c + 10a^2b^4c - 80a^3b^3c - 4a^3b^3c
\]

\[
U^{III}_n - U^{IV}_n = \frac{A^2 ab \Delta_5}{6 (a + b + 2c)^2 (3a + 8ac + 8bc + 16c^2)^2} \leq 0
\]

\[
\Delta_5 = 4a^3b^2 - 5a^3b - 16a^3c + 320bc^3 + 16b^3c - a^2b^2 - 96a^2c^2 + 128b^2c^2 + 256c^4 + 48abc^2 + 32ab^2c - 36a^2bc
\]

\[
U^{III}_m - U^{IV}_m = \frac{A^2 ab \Delta_6}{6 (3ab + 8ac + 8bc + 16c^2)^2 (a + b + 2c)^2} \leq 0
\]

\[
\Delta_6 = -5ab^3 + 4a^3b + 320ac^3 + 16a^3c - 16b^3c - a^2b^2 + 128a^2c^2 + 96b^2c^2 + 256c^4 + 48abc^2 - 36ab^2c + 32a^2bc
\]

\[
2U^I_{in} - U^{III}_{in} = \frac{A^2 a \Delta_7}{3 (3ab + 8ac + 8bc + 16c^2)^2 (3a + 3b + 4c)^2} \leq 0
\]

\[
\Delta_7 = -9ab^4 - 256b^4c - 512bc^4 - 36b^4c + 9a^3b^2 - 192a^2c^3 - 16a^3c^2 - 64ab^2c^2 - 256b^2c^2 - 512ab^3c + 24a^3bc - 256ab^2c^2 + 24a^2b^2c
\]

\[
2U^I_{im} - U^{III}_{im} = \frac{2A^2 b \Delta_8}{3 (3ab + 8ac + 8bc + 16c^2)^2 (3a + 3b + 4c)^2} \leq 0
\]

\[
\Delta_8 = -9a^4b - 36a^4c - 72a^3bc - 256a^3c^2 + 9a^3b^3 + 24a^2b^2c - 256a^2bc^2 - 640a^2c^3 + 24ab^3c - 512abc^3 - 512ac^4 - 16b^3c^2 - 192b^2c^3 - 256bc^4
\]

3. For \( a > b \) \( U^I_n > U^I_m \) and \( U^I_n > U^{IV}_m \), analog for \( b > a \) \( U^I_n < U^I_m \) and \( U^I_n < U^{IV}_m \), if \( a = b \)
$U_{IV}^n = U_n^I.$

\[
U_{IV}^n - 2U_{in}^I = \frac{A^2 a (a - b) \Delta_9}{6(2ab + 3ac + 3bc + 4c^2)^2 (a + b + 2c)^2} > 0
\]

\[
\Delta_9 = 2ab^3 + 12ac^3 + 36bc^3 + 4b^3c + 2a^2b^2 + a^2c^2 + 23b^2c^2 + 16c^4 + 28abc^2
\]

\[
+ 16ab^2c + 4a^2bc
\]

\[
U_m^I - 2U_{im}^I = \frac{A^2 a (b - a) \Delta_{10}}{6(2ab + 3ac + 3bc + 4c^2)^2 (a + b + 2c)^2} > 0
\]

\[
\Delta_{10} = 2a^3b + 36ac^3 + 4a^3c + 12bc^3 + 2a^2b^2 + 23a^2c^2 + b^2c^2 + 16c^4 + 28abc^2
\]

\[
+ 4abc^2 + 16a^2bc
\]

### B.5 Proposition 6

The ranking of welfare $W^{II} > W^I \geq W^{IV} > W^{III}$ can be established through subscription of welfare terms.

\[
W^{II} - W^I = \frac{2A^2 ab(a + b + 4c) \Lambda_1}{9(3a + 3b + 4c)^2 (2ab + 3ac + 3bc + 4c^2)^2} > 0
\]

\[
\Lambda_1 = 11ab^2 + 11a^2b + 32ac^2 + 15a^2c + 32bc^2 + 15b^2c + 16c^3 + 42abc
\]

\[
W^I - W^{IV} = \frac{A^2 c(a - b)^2 \Lambda_2}{18(2ab + 3ac + 3bc + 4c^2)^2 (a + b + 2c)^2} > 0
\]

\[
\Lambda_2 = 8ab^2 + 8a^2b + 28ac^2 + 11a^2c + 28bc^2 + 11b^2c + 16c^3 + 34abc
\]

\[
W^{IV} - W^{III} = \frac{A^2 ab(a + b + 8c) \Lambda_3}{18(a + b + 2c)^2 (3ab + 8ac + 8bc + 16c^2)^2} > 0
\]

\[
\Lambda_3 = 13ab^2 + 13a^2b + 96ac^2 + 32a^2c + 96bc^2 + 32b^2c + 64c^3 + 84abc
\]